

## **ERROR DIAGNOSIS OF PROBABILITY OF OUTER FINITE POTENTIAL WELL THEORY USING RANDOM SIGNAL ANALYSIS**

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### **ABSTRACT**

Wireless networking is the branch of communication in which wireless channels are used for the transmission of the signals. There are various terminologies in the field of wireless networking which are taken from quantum physics. In fact quantum physics consists of the signals which are in wireless medium. There are many important theories which are given in the field of quantum mechanics which can be related with the concepts of wireless networks. Some of them are directly or indirectly related to wireless networks. There is always a possibility of an error in the boundary conditions which can be found out by the mean of Random signal analysis. Also the major problem in the communication and wireless network is probability error. Many theories are provided to reduce this error.

**KEYWORDS:** Momentum, Wavelets, Duality, Uncertainty

### **1. INTRODUCTION**

Quantum Physics is the branch of science which highly deals with the electrons and other subatomic particle behaving as in the wave nature. Generally, scientists have two views that electron and other subatomic particles has two nature either wave or particle. Several theories were presented by the scientists to prove that electron follows wave nature. Generally in the field of wireless networking, waves are used for the transmission of the signal. The waves which are used can be compared to the electron and the other sub-atomic particle waves as generated by the wave nature duality.

In the similar way various theories are implemented in the field of quantum physics can be implemented in the field of wireless network. In this paper we have tried to implement the use of the Schrodinger Principle in the field of wireless networking. The paper includes the error diagnosis of probability in finite potential well model using random signal analysis. The paper also includes the concepts the probability in which the normal distribution of probability is used for signal analysis. Probability of the energy density is studied between the nodes by considering cases of boundary condition. Error in the probability will be derived by considering the Heisenberg and Schrödinger equation. Normally in normal form of distribution, the step deviation for which a constant probability occurs but generally there is an error in the probability which can be linked to the error in the position, velocity or momentum of the particle.

Heisenberg's uncertainty principle is a special case and it refers to wave packets with Gaussian distribution. The cases of Schrödinger equations which include the energy quantization, boundary conditions are taken from [1]. The proof of the strong inequality was given by Kennard and Weyl [2]. Later Robertson [3] generalized the correlation for

arbitrary observables  $A$  and  $B$  [1], and Dichburn [4] presented the relation between Heisenberg's fluctuations. Generalized and précised form of Heisenberg's principle was given by Schrödinger [5, 6] and Robertson [7]. The Schrödinger's relation [3] can be expressed in a compact [6] and "very elegant form" through [9]. Ref. [8] explains about the role of noise in increasing the error. Ref [10] gives the information about the classical cases of quantum analogy used in handbooks on experimental physics. Ref. [11] gives the information regarding the matrix analogy for quantum physics. Definition of quantum mechanics is taken from [12]. There are two cases of Schrödinger equation as time independent and other as time independent, we take ref. [13] in which the probability distribution of both the cases are studied

The remainder of the paper is described in the following ways: Section II gives the brief introduction regarding the related work. In the section III, approximation in the error in probability with respect to energy and position is defined and derived by considering the finite potential well wave equations of Schrödinger wave equation. Also some further research and problems are explained in section IV. Finally conclusion is in section V.

## 2. RELATED THEORIES

- **Schrödinger Wave Equation**

In the quantum mechanics, electron is characterized with a three dimensional wave function  $\psi$ . This is a wave equation popularly known as Schrodinger Wave Equation which is based on the equations based on the elements of Newton's classical mechanics and wave- particle duality.

$$\nabla^2 \psi + \frac{8\pi^2 m}{h^2} (E - V) \psi = 0$$

Here  $\nabla^2$  is the del operator (laplacian operator).

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

Here  $E$  is the total energy of the system and  $V$  is the potential energy of the system.

However, there are two conditions which satisfy the Schrödinger wave equations, one which follows the boundary conditions and the other are the scattering conditions.

- **Boundary Conditions**

In the case of Boundary conditions, bondage is provided over the potential energy of the signal, outside a fixed boundary the probability of the energy density is zero and there is an increase in the probability as the radius of the fixed boundary decreases. Therefore if we have a single particle of mass  $m$  confined to within a region  $0 < x < L$  with potential energy  $V = 0$  bounded by infinitely high potential barriers, i.e. the potential experienced by the particle is then:

$$V(x) = 0 \quad 0 < x < L$$

$$= \infty \quad x \geq L, x \leq 0$$

In the regions for which the potential is infinite, the wave function will be zero, that is, there is zero probability of the particle being found in these regions. Thus, we must impose the boundary conditions

$$\psi(0) = \psi(L) = 0.$$

- **Scattering over Potential Barrier**

It has been derived that energy waves has the tendency to lose their energy while travelling from one node to another but time required for losing their complete energy is infinite. Though after a certain limit of distance the energy can be compared to a nil but it usually has exponential decrement pattern. Actually when there is a connection between two nodes then there is probability difference and a respective energy gap present so that the electron wave in the form of signal move from one node to another. Therefore we can say that there is a particular threshold voltage over a certain point (here  $x=0$ ) which has been compared with the signal voltage and if the signal voltage is less, then the energy signal will be reverse its direction from  $x=0$ . Voltage distribution of scattering condition is given below

$$V(x) = 0 \quad x < 0$$

$$= V \quad x > 0$$

By studying the voltage distribution, probability distribution will be

$$\psi(x) \rightarrow 0 \text{ as } x \rightarrow \pm\infty$$

Extending the concepts of Heisenberg Uncertainty principle there is a chance of having an uncertainty in the presence of connection between two nodes. As discussed the connection between two nodes depends on the energy density of the signals present between these signals. The change of this probability is dependent on the value of  $\Delta E$  as the value of  $p$  is related with that of energy gradient. The cutoff value which is required for the connection between the two nodes is shown below:

$$\Delta p \cdot \Delta E \geq c'$$

Here  $c'$  is a constant whose value can be calculated form the practical experiments.

Consider the case of standing wave generation in one dimension in which there are two nodes which are connected through a string. When an oscillation is provided to this system it has the tendency to generate a standing wave at the ends of the nodes (starting and ending point). This example can be compared with the three dimensional wireless network where if two nodes are connected to each other then they will have wavelets having oscillations along their path which generates travelling waves at the connecting path of both the nodes but at these ends there is a formation of standing waves. According to the property of standing waves, there is no interference among the signals. This acts as a plus point for the wireless networks as this leads to no cross connection of the signals within the nodes (this is the basic concept which is generally used for zero interference within the wavelets).

Considering the following case, the potential experienced by the particle in finite potential well is

$$V(x) = 0 \quad 0 < x < L$$

$$= V_0 \quad x \geq L, x \leq 0$$

Now to solve the time independent Schrödinger wave equation for the following range of potentials,

$$\frac{h^2}{4\pi m} \nabla^2 \Psi(x) + (E - V)\psi(x) = 0; \quad x \leq 0$$

$$\frac{h^2}{4\pi m} \nabla^2 \Psi(x) + E\psi(x) = 0; \quad 0 < x < L$$

$$\frac{h^2}{4\pi m} \nabla^2 \Psi(x) + (E - V)\psi(x) = 0; x \geq 0$$

In order to get second order differential equation

We assume  $V > E$ ,

$$\alpha = \sqrt{\frac{8\pi^2 m E}{h^2}} \quad \beta = \sqrt{\frac{8\pi^2 m (V - E)}{h^2}}$$

Here  $\alpha$  and  $\beta$  are real numbers. Now the equations can be written as:

$$\nabla^2 \Psi(x) - \alpha^2 \psi(x) = 0; x \leq 0$$

$$\nabla^2 \Psi(x) + \beta^2 \psi(x) = 0; 0 < x < L$$

$$\nabla^2 \Psi(x) - \alpha^2 \psi(x) = 0; x \geq L$$

Considering the first equation, the solutions to this equation be

$$\psi(x) = A e^{-\alpha x} + B e^{\alpha x}$$

Here A and B are unknown constants. At this point we can use the boundary conditions as when  $x \rightarrow \infty$ ,  $\psi(x)$  tends to 0. Since  $x \leq 0$  the value of  $\psi(x)$  at  $x = -\infty$ . It will diverge the system, to avoid this  $A = 0$ . Therefore we have

$$\psi(x) = B e^{\alpha x} \quad x \leq 0$$

Similarly take the second case where  $0 < x < L$  therefore take the general solution for this equation.

$$\psi(x) = P \cos(Mx) + Q \sin(Mx)$$

Consider the third case  $x \geq 0$ , in this case we have the same condition as in case one as when  $x \rightarrow \infty$ ,  $\psi(x)$  tends to 0. Therefore the solution for this equation

$$\psi(x) = C e^{-\alpha x} \quad x \geq L$$

Now according to the property that the wave which occurs should be continuous in nature, therefore the three equations should be continuous at all their removable discontinuity.

Therefore, on analyzing on I equation and II equation we get

$$B = P$$

$$\alpha B = \beta Q$$

On analyzing the value of II and III at  $x = L$ , by differentiating the equation.

$$C e^{-\alpha L} = P \cos(\beta \cdot L) + Q \sin(\beta \cdot L)$$

On differentiating

$$-\alpha C e^{-\alpha L} = -\beta P \sin(\beta \cdot L) + \beta Q \cos(\beta \cdot L)$$

Converting this equation to the matrix form

$$\begin{pmatrix} \alpha & -\beta \\ \alpha \cos(\beta \cdot L) - \beta \sin(\beta \cdot L) & \alpha \cos(\beta \cdot L) + \beta \sin(\beta \cdot L) \end{pmatrix} \begin{pmatrix} P \\ Q \end{pmatrix}$$

The above matrix is equalized to zero. To get a non-trivial solution of the above equation, the modulus of the (2x2) matrix should be zero. On equalizing and simplifying we get:

$$\psi(x) = \begin{cases} B e^{\alpha x}, & x \leq 0 \\ B(\cos \beta x + \frac{\alpha}{\beta} \sin \beta x), & 0 < x < L \\ B e^{-\alpha(x-L)}, & x \geq L \end{cases}$$

On observing the value of  $\psi(x)$  from  $x \leq 0$  and  $x \geq L$ , it is found that the value is delayed by  $e^{\alpha L}$ . Therefore the value of change in probability for first case will be proportional to last case and the proportionality constant will be  $e^{\alpha L}$  to find the value of constant B, we use the following theorem.

$$\int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1$$

Therefore, applying the above equation:

$$|B|^2 [\int_{-\infty}^0 e^{2\alpha x} dx + \int_0^L (\cos \beta x + \frac{\alpha}{\beta} \sin \beta x)^2 dx + \int_L^{\infty} e^{-2\alpha(x-L)} dx] = 1$$

By calculations we get

$$B = \frac{\beta}{\beta_0} \sqrt{\frac{\alpha}{\frac{\alpha L}{2} + 1}}$$

Where  $\beta_0 = \sqrt{\frac{8\pi^2 m V}{h^2}}$

By solving the above equation we have

$$P_{out} = |B|^2 [\int_{-\infty}^0 e^{2\alpha x} dx + \int_L^{\infty} e^{-2\alpha(x-L)} dx] = |B|^2 \alpha^{-1}$$

As discussed above we will consider the change in probability due to the outer potential wall.

$$\psi(x) = \begin{cases} B e^{\alpha x}, & x \leq 0 \\ B e^{-\alpha(x-L)}, & x \geq L \end{cases}$$

Now consider the normal distribution of the probability then the probability function will be as following:

$$\psi(x)_{env} = \sqrt{P_x} = \sqrt{A_x} e^{-\frac{x-x_0}{4\sigma_x^2}}$$

$$B e^{\alpha x} = \sqrt{A_x} e^{-\frac{x-x_0}{4\sigma_x^2}}$$

On comparing the values of both the functions, we get the following result:

$$B = \sqrt{A_x}$$

$$e^{\alpha x} = e^{-\frac{x-x_0}{4\sigma_x^2}}$$

Taking natural log on both sides

$$\alpha x = \frac{x_0 - x}{4\sigma_x^2}$$

$$4\sigma_x^2 = \frac{x_0}{\alpha x} - \frac{1}{\alpha}$$

Now differentiating this equation with respect to x, therefore

$$8\sigma_x \cdot \Delta\sigma_x = -\frac{x_0}{\alpha x^2} \cdot \Delta x$$

This relation describes the relation between the step deviation error and error in distance. The relation between the error in position and error in probability is also described below.

$$P = |B|^2 \alpha^{-1}$$

$$\Delta P = 0$$

It is independent of x therefore change in probability with respect to change in x is 0. Also the change in probability with respect to change in  $\Delta\sigma_x$  is also nil. Since  $\Delta P = 0$ , relations which are developed by the Heisenberg principle are satisfied with a zero condition as

$$\Delta P = d \cdot \Delta x$$

$$\Delta x \cdot \Delta p \geq c/4\pi$$

$$\Delta x \cdot \Delta v \geq c/4\pi m$$

On modifying the equations we have,

$$\Delta P \cdot \Delta p \geq c \cdot d/4\pi$$

$$\Delta P \cdot \Delta v \geq c \cdot d/4\pi m$$

Since  $\Delta P = 0$ ,  $d=0$ ; the above conditions are valid for zero change in probability. The probability will be constant for the changes in the value of position, momentum.

Also individual probabilities of both outer section of the finite potential well are independent of x, therefore the individual probability error are also nil. Due to this the change in momentum or change in position has no effect over the change of probability.

## APPICATION

This area where the value of probability error is nil can be used for the communication field, wireless networks etc. As probability error is nil, this area is not affected by the change in position of energy packet. Probability error is a serious issue in communication networks which can be solved by the use of this area for communication field.

## CONCLUSIONS

The paper consist a brief introduction about the error in the probability conditions in the finite potential well boundary conditions of Schrödinger equation. From the given equations, probability is derived and it is clear that the probability error in the region  $x < 0$  and  $x > L$  is zero. Due to which probability error won't be changed for any change in position, momentum.

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